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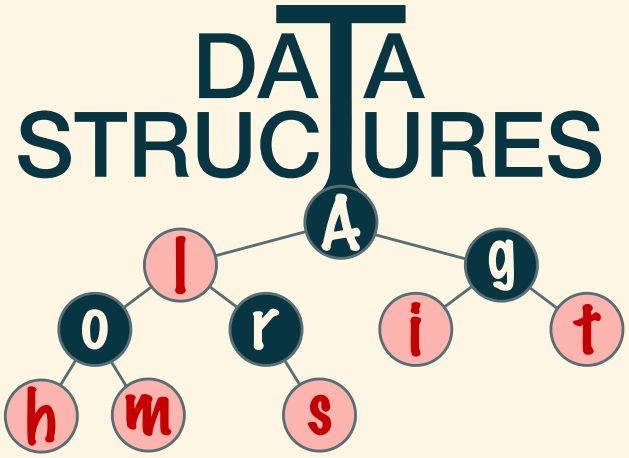

WEEK 11: DYNAMIC PROGRAMMING (LAB)

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LEARNING OBJECTIVES

By the end of this lab session, you should be able to:

- Implement recursive and dynamic programming solutions to problems.
- Understand the time complexity differences between naive recursion and optimized DP approaches.
- Solve algorithmic problems such as Mobile Game Level Progression, DNA Sequence Analysis, and Vending Machine Programming.

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OVERVIEW OF LAB ACTIVITIES

Here are the activities for which you will upload solutions to xSITe or Gradescope:

1. **Mobile Game Level Progression - Recursive**: Submit using Gradescope. Total 25 marks.
2. **Mobile Game Level Progression - Iterative (DP)**: Submit using Gradescope. Total 25 marks.
3. **DNA Sequence Analysis - Longest Common DNA Subsequence**: Submit using Gradescope. Total 25 marks.
4. **Vending Machine Programming - Optimal Coin Change**: Submit using Gradescope. Total 25 marks.

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EXERCISE 1A: MOBILE GAME LEVEL PROGRESSION - RECURSIVE

TASK

You are developing a progression system for a popular mobile RPG game called **"Level Nexus"**. In this game, players can advance through levels by completing quests or challenges. The game mechanics allow a player to advance either `1` or `2` levels at a time (depending on quest difficulty they choose).

Given that a player wants to reach level `n`, your task is to calculate the number of distinct progression paths they can take to reach that level from level 0. **Code a recursive solution to this problem. Avoid using loops or arrays.**

EXAMPLES

```
Input: n = 2
Output: 2
Explanation: (1 level + 1 level) OR (2 levels at once)

Input: n = 3
Output: 3
Explanation: (1 level + 1 level + 1 level) OR (1 level + 2 levels) OR (2 levels + 1 level)
```

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EXERCISE 1A: MOBILE GAME LEVEL PROGRESSION - RECURSIVE

CONSTRAINTS

The parameter `n` can take values between 1 and 45 (inclusive), representing the target level.

TEMPLATE

You can find the template files on xSITe. Use the provided `makefile` to compile and run your project.

SUBMISSION

Implement the `calculateProgressionPaths` function using recursion. Submit your completed `mobile_game_progression_recursive.cpp` file to the Gradescope assignment titled **Week 11 Lab Exercise 1A: Mobile Game Level Progression - Recursive**.

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EXERCISE 1B: MOBILE GAME LEVEL PROGRESSION - ITERATIVE (DP)

TASK

Continuing with the **"Level Nexus"** mobile game, you now need to optimize the progression path calculation system. The game's backend needs to handle thousands of concurrent players checking their possible progression routes efficiently.

Given that a player wants to reach level n, calculate the number of distinct progression paths they can take (advancing 1 or 2 levels at a time).

REQUIREMENTS

Give a bottom-up dynamic programming solution to this problem (tabulation). You are encouraged to use arrays. Do NOT use recursion for this problem.

EXAMPLE

```
Input: n = 4
Output: 5
Explanation: (1+1+1+1) OR (2+1+1) OR (1+2+1) OR (1+1+2) OR (2+2)
```

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EXERCISE 1B: MOBILE GAME LEVEL PROGRESSION - ITERATIVE (DP)

CONSTRAINTS

The parameter `n` can take values between 1 and 45 (inclusive), representing the target level.

TEMPLATE

You can find the template files on xSITe.

Implement the `calculateProgressionPathsDP` function using **dynamic programming (bottom-up)**. Use either an array-based approach (`O(n)` space) or an optimized two-variable approach (`O(1)` space).

SUBMISSION

Submit your completed `mobile_game_progression_dp.cpp` file to the Gradescope assignment titled **Week 11 Lab Exercise 1B: Mobile Game Level Progression - Iterative (DP)**.

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EXERCISE 2: LONGEST COMMON DNA SUBSEQUENCE

TASK

You are working as a bioinformatics researcher studying **evolutionary relationships** between different organisms. Your task is to analyze DNA sequences from two different species to find their **longest common DNA subsequence**.

This helps scientists understand how closely related these organisms are - the longer the common DNA sequence, the more recent their evolutionary split.

Given two DNA strings `dna1` and `dna2` (containing only characters A, C, G, T), your task is to return **the length of their longest common subsequence (LCS).**

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EXERCISE 2: LONGEST COMMON DNA SUBSEQUENCE

DEFINITIONS

A **DNA subsequence** is a sequence that can be derived from the original DNA sequence by deleting some nucleotides without changing the order of the remaining nucleotides.

For example, from the DNA sequence **ACGTACGT**, valid subsequences include **ACG**, **AGCT**, **ACGT**. Sequences like *TGCA* or *CGTA* are **NOT** valid subsequences (order changed).

A **common DNA subsequence** between `dna1` and `dna2` is a subsequence that appears in both DNA strings. For example, the DNA subsequence **ACTG** might appear in both **ACGTACGT** and **ACTGTCCG**.

The **longest common DNA subsequence** between `dna1` and `dna2` is the common subsequence that has the maximum length, indicating the highest degree of evolutionary similarity.

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EXERCISE 2: LONGEST COMMON DNA SUBSEQUENCE

EXAMPLE INPUT AND OUTPUT

```
Input: dna1 = "ACGTACGT", dna2 = "ACTTGCGT"
Output: 6  // longest common subsequence is "ACTCGT"

Input: dna1 = "ATCGATCG", dna2 = "TCGAATCG"
Output: 7  // longest common subsequence is "TCGATCG"

Input: dna1 = "AAAA", dna2 = "TTTT"
Output: 0  // No common subsequence between these DNA sequences
```

TEMPLATE

You can find the template files on xSITe. Implement the `longestCommonSubsequence` function using **dynamic programming** with a 2D table. This table should store the length of the LCS for different prefixes of the input DNA sequences.

SUBMISSION

Submit your completed `longest_common_subsequence.cpp` file to the Gradescope assignment titled **Week 11 Lab Exercise 2: DNA Sequence Analysis - Longest Common DNA Subsequence**.

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EXERCISE 2: LONGEST COMMON DNA SUBSEQUENCE

HINTS

- Consider the prefixes of the DNA sequences `dna1` and `dna2`. The longest common subsequence of the `dna1` and `dna2` depends on **the longest common subsequence of their prefixes**.
- Use a 2D table to store the solutions to the subproblems instead of recursion. A table called **LCS(i,j)** should store the longest common subsequence of the prefix of `dna1` with length `i` and the prefix of `dna2` with length `j`.
- **If the last nucleotides of the prefixes match** (same DNA base), include them in the LCS and move to the previous nucleotides: `LCS(i,j)=LCS(i−1,j−1)+1`.
- **If the last nucleotides don't match**, try removing one nucleotide at a time and take the maximum result: `LCS(i,j) = max(LCS(i−1,j),LCS(i,j−1))`.

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EXERCISE 2: LONGEST COMMON DNA SUBSEQUENCE

HINTS

$$LCS(i,j) = \begin{cases} 0, & \text{if } i = 0 \text{ or } j = 0 \\ LCS(i-1, j-1) + 1, & \text{if } dna1[i-1] = dna2[j-1 \\ \max(LCS(i-1,j), LCS(i,j-1)), & \text{if } dna1[i-1] \neq dna2[j-1 \end{cases}$$

- The final solution is `LCS[m][n]`, where `m` is the length of `dna1` and `n` is the length of `dna2`.

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EXERCISE 3: VENDING MACHINE PROGRAMMING - OPTIMAL COIN CHANGE

TASK

You are programming the **change dispensing system** for a modern smart vending machine. The vending machine has a limited set of coin denominations available in its coin hopper, and your goal is to **minimize the number of coins dispensed** as change to customers.

Given an integer array `coins` representing the available coin denominations in the vending machine and an integer `amount` representing the change amount to be dispensed, return **the fewest number of coins needed to make up that amount**.

NOTE: If the exact change amount cannot be made using the available coin denominations, return -1.

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EXERCISE 3: VENDING MACHINE PROGRAMMING - OPTIMAL COIN CHANGE

EXAMPLES

```
Input: coins = {1,2,5}, change amount = 11
Output: 3  // Dispense (5¢ + 5¢ + 1¢)
Explanation: The vending machine can dispense 11¢ change using only 3 coins: 5¢ + 5¢ + 1¢. Other
combinations like (5¢ + 2¢ + 2¢ + 2¢) use more coins and are less efficient.

Input: coins = {2}, change amount = 3
Output: -1
Explanation: The vending machine only has 2¢ coins available, so it cannot make exactly 3¢ change. In
your program, you should return -1. In real life, the machine should display "exact change only".
```

TEMPLATE

You can find the template files on xSITe.

Implement the `coinChange` function inside the partial `coin_change.cpp` file. Use a **dynamic programming** approach to solve this problem efficiently.

SUBMISSION

Submit your completed `coin_change.cpp` file to the Gradescope assignment titled **Week 11 Lab Exercise 3: Vending Machine Programming - Optimal Coin Change**.

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EXERCISE 3: VENDING MACHINE PROGRAMMING - OPTIMAL COIN CHANGE

HINTS

- Think in terms of choices. For each coin denomination, the vending machine can choose to dispense that coin or skip it.
- For each subproblem (`amount = 1`, `amount = 2` ... `amount = target`), try all available coin choices and select the one that uses the fewest total coins.
- Let `dp(amount)` represent the minimum number of coins needed to make change for that amount. The recurrence relation is: `dp(amount) = 1 + min(dp(amount-c))` for all coin denominations `c` available in the vending machine.
- If no valid combination exists for the target amount, return `-1` (machine cannot provide exact change).

CONCLUSION

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WRAP-UP

By the end of this lab you should be able to:

1. **Understand dynamic programming fundamentals**: Distinguish between recursive and dynamic programming approaches, and recognize when DP optimization is beneficial
2. **Implement simple top-down solutions**: Write clean recursive algorithms and analyze their exponential time complexity
3. **Apply bottom-up techniques**: Implement tabulation (bottom-up) for problems solvable using dynamic programming
4. **Solve real-world optimization problems**: Apply DP to mobile game progression systems, bioinformatics DNA analysis, and vending machine programming

OUTLOOK

This lab covered **Dynamic Programming** as a powerful optimization technique for solving complex problems with overlapping subproblems. The remaining weeks will explore:

Greedy Algorithms

Local optimization strategies that build globally optimal solutions step-by-step