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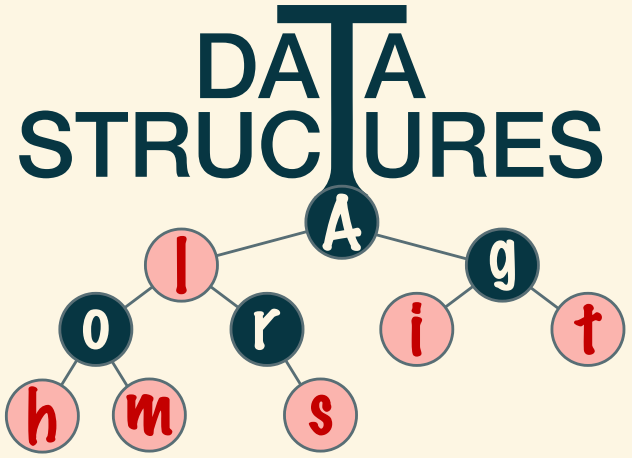

WEEK 10: SINGLE-SOURCE SHORTEST PATHS (LAB)

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INTRO

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LEARNING OBJECTIVES

By the end of this lab session, you should be able to:

1. **Understand single-source shortest path algorithms:** Explain how Dijkstra's (priority queue, greedy) and Bellman-Ford (relaxation iterations) work, compare their time complexities, and identify when to use each based on graph properties (negative weights, cycles).
2. **Implement Dijkstra's algorithm:** Implement Dijkstra's in C++ using priority queues and maps, handle string-based vertex names with adjacency lists, track predecessors, and detect negative edge weights.
3. **Apply shortest path algorithms:** Use Bellman-Ford to solve pathfinding problems in game development and model grid-based problems as graphs to find optimal paths.

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OVERVIEW OF LAB ACTIVITIES

This lab consists of three main activities:

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ACTIVITY 1: MULTIPLE CHOICE QUESTIONS

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EXERCISE: UNDERSTANDING DIJKSTRA'S AND BELLMAN-FORD ALGORITHMS

This activity consists of multiple choice questions to test your understanding of **single-source shortest path algorithms**, specifically **Dijkstra's algorithm** and **Bellman-Ford algorithm**.

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INSTRUCTIONS

- Complete the xSITe quiz questions covering Dijkstra's and Bellman-Ford algorithms
- Questions may include:
  - Selecting the correct algorithm for a given scenario
  - Identifying the order of vertex processing
  - Calculating shortest distances after specific iterations
  - Understanding time complexity and space complexity
  - Recognizing when negative cycles exist

Submission (xSITe Quizzes, 10 marks). Complete the quiz "Week 10 Lab Exercise 1" on xSITe.

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KEY CONCEPTS TO REVIEW

Before attempting the quiz, make sure you understand:

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ACTIVITY 2: SINGLE-SOURCE SHORTEST PATH WITH DIJKSTRA'S ALGORITHM

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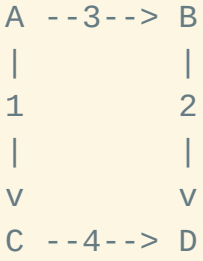
PROBLEM DESCRIPTION

You are given a **weighted directed graph** represented as an adjacency list, where each vertex is identified by a string name. Your task is to implement **Dijkstra's algorithm** to find the **shortest distances** from a given source vertex to all other vertices in the graph.

This is a classic **single-source shortest path** problem where:

- Each vertex is identified by a string (e.g., "A", "B", "start", "end")
- Edges have non-negative weights
- We need to find the shortest path from a source vertex to all reachable vertices

Consider the following graph:



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HYPOTHETICAL SCENARIO: LOGISTICS ROUTING IN NORTHERN SINGAPORE (SHORTEST PATH CALCULATION)

A courier hub in **Woodlands (A)** needs to deliver a parcel to a customer in **Bukit Panjang (D)**. The values on the map represent the actual distance in **kilometers (km)** between the locations.

Using the starting point of **Woodlands (A)**, calculate the shortest distance to each town:

- **Distance to Woodlands (A): 0 km** (Origin)
- **Shortest distance to Yishun (B): 3 km** (Direct via Woodlands (A) → Yishun (B))
- **Shortest distance to Kranji (C): 1 km** (Direct via Woodlands (A) → Kranji (C))
- **Shortest distance to Bukit Panjang (D): 5 km** (path A → C → D with cost 1 + 4 = 5, or A → B → D with cost 3 + 2 = 5)

Result: In this specific graph, both the Yishun route and the Kranji route result in a total distance of **5 km**.

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CONSTRAINTS

- $1 \leq |V| \leq 1000$, where $|V|$ is the number of vertices
- $0 \leq |E| \leq 10000$, where $|E|$ is the number of edges
- Edge weights are integers (typically non-negative, but **negative weights may appear in test cases** — since Dijkstra's greedy approach assumes no negative edges, your implementation must detect them and return an empty map to indicate the input is invalid for this algorithm).
- Vertex names are strings (alphanumeric)
- The graph may be disconnected (some vertices may be unreachable from the source)

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EXERCISE

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EXERCISE (CONT.)

4. **Implementation requirements**:
    - Use **Dijkstra's algorithm** with a priority queue (min-heap)
    - Initialize all distances to `INT_MAX` except the source (distance = 0)
    - Track visited vertices to avoid reprocessing
    - Update `predecessors` map when relaxing edges
    - Handle the case where negative edge weights are detected (return empty map)
    - Attached are 4 Test files to test your algorithm.

Submission (Gradescope, 20 marks). Submit your completed `dijkstra.cpp` to the "Week 10 Lab Exercise 2" assignment on Gradescope.

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KEY POINTS & IMPLEMENTATION DETAILS

- **Initialization:** All distances = `INT_MAX`, source = 0, all vertices unvisited, predecessors = empty strings.
- **Priority Queue:** Use `std::priority_queue` with `std::greater` for min-heap. Stores `(distance, vertex_name)` pairs, always processes minimum distance first.
- **Relaxation:** For each edge, check negative weights (return empty map if found). If `dist[current] + weight < dist[neighbor]`, update distance and predecessor, push to queue.
- **Lazy Deletion:** Multiple entries per vertex may exist in queue; skip already visited vertices to avoid update/remove operations.
- **Edge Cases:** Negative weights return empty map; unreachable vertices remain `INT_MAX`; disconnected graphs handled naturally.

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ACTIVITY 3: GAME PATHFINDING WITH SHORTEST PATH ALGORITHMS

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PROBLEM DESCRIPTION

You are designing a **pathfinding system** for a 2D grid-based game. The game world consists of a grid where each cell has a **terrain cost** (movement cost). The player character starts at position (s_r, s_c) and needs to reach the target position (t_r, t_c).

The character can move in **4 directions** (up, down, left, right) to adjacent cells. The cost of moving from one cell to an adjacent cell is the **terrain cost of the destination cell**. Your task is to find the **minimum total cost** to move from the start to the target.

This is a classic **single-source shortest path** problem where:

- Each grid cell is a **vertex**
- Edges connect adjacent cells (4-neighbour connectivity)
- Edge weights are the terrain costs of destination cells
- We need to find the shortest path from source (s_r, s_c) to target (t_r, t_c)

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PROBLEM DESCRIPTION (CONT.)

Consider a 3×4 game grid with terrain costs:

```
   0  1  2  3
 ------------
0| 1  5  2  1
1| 3  1  4  2
2| 1  2  1  3
```

Starting at $(0, 0)$ with cost 1, and target at $(2, 3)$ with cost 3.

Example path: $(0, 0) \to (0, 1) \to (1, 1) \to (2, 1) \to (2, 2) \to (2, 3)$

Path cost: $1 + 5 + 1 + 2 + 1 + 3 = 13$ (includes start cell cost)

Note: The optimal path might be different! Use Bellman-Ford to find it.

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CONSTRAINTS

- $1 \leq rows, cols \leq 100$
- $0 \leq \text{terrain\_cost}[i][j] \leq 1000$
- Start and target positions are always valid (within grid bounds)
- The grid is guaranteed to be connected (all cells are reachable)

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EXERCISE

1. **Download:** Get `game_pathfinding.zip` from xSITe. The starter file `game_pathfinding.cpp` contains the stub:

```cpp
int shortestPathCost(int rows, int cols,
                     const vector<vector<int>> &terrain,
                     int start_r, int start_c,
                     int target_r, int target_c);
```

2. **Input format**:
   - First line: `rows cols`
   - Next line: `start_r start_c target_r target_c`
   - Next `rows` lines: each line has `cols` integers representing terrain costs
3. **Output format**: Return the minimum total cost to move from start to target (including the cost of the start cell).

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EXERCISE (CONT.)

4. **Implementation requirements**:
    - Use **Bellman-Ford algorithm**
    - Map (row, col) to a 1D index: `idx = row * cols + col`
    - Handle 4-directional movement (up, down, left, right)
    - Use appropriate data structures (using relaxation for Bellman-Ford)
    - Attached are 3 test files for testing your algorithm.

Submission (Gradescope, 30 marks). Upload completed `game_pathfinding.cpp` to Gradescope "Week 10 Lab Exercise 3".

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KEY POINTS

- **Bellman-Ford:** Relax all edges $(|V| - 1)$ times. Slower but handles negative weights and detects cycles.
- **Graph:** Vertices = grid cells (row, col); edges = 4-neighbour connections; edge weight = terrain cost of destination cell.

CONCLUSION

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WRAP-UP

By the end of this lab you should be able to:

1. Construct graphs for different use cases
2. Use appropriate data structures to implement single source shortest paths algorithms
3. Implement **Dijkstra's or Bellman-Ford algorithm** to find shortest paths from a source vertex
4. Apply graph algorithms to solve real-world problems (shortest pathfinding in games)

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OUTLOOK

This lab covered both Minimum Spanning Trees and Single-Source Shortest Paths as fundamental graph algorithms. The remaining weeks cover:

Dynamic Programming and Greedy Algorithms

Versatile optimization techniques for solving complex problems